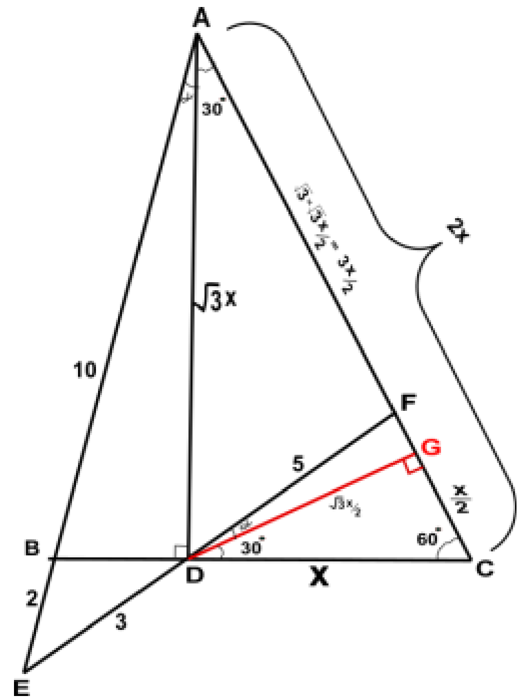
I PRIZE WINNER MRS.MADHUMITHA'S SOLUTION



Given ABC is a Δ, AD Altitude

$\therefore \angle ADC = 90°$, $\angle DAC = 30°$ (given)

$\therefore \angle ACD = 60°$. ΔADC is special triangle

Let DC = x unit, AC= 2x unit AD = $\sqrt{3x}$ unit ----------------(I)

Construction:

Choose a point 'G' in AC such that DG ⊥ AC

$\therefore \angle GDC = 30°$ $(\because \angle DCG = 60°)$

In Δ ADB, Let $\Delta DAB = \alpha$

Consider the Δ ABD & Δ DFG.

As ΔDGC is a special triangle [$\angle DGC = 90°$, $\angle GDC = 30°$]

$\therefore CD = x$, GC = $\frac{x}{2}$, GD = $\frac{\sqrt{3}x}{2}$.

Now $\frac{AB}{FD} = \frac{10}{5} = 2$ ----------------- (1)

$\frac{AD}{DG} = \frac{\sqrt{3}x}{\frac{\sqrt{3}x}{2}} = 2$ -------------- (2)

FG = $\sqrt{25 - \frac{3x^2}{4}}$ = $\frac{\sqrt{100-3x^2}}{2}$ (ΔFGD right angled at G, DG⊥ AC)

And BD = $\sqrt{100 - 3x^2}$

$\Rightarrow \frac{BD}{FG}$ = $\frac{\sqrt{100-3x^2}}{\frac{\sqrt{100-3x^2}}{2}}$ = 2 ----------------- (3)

(1), (2) & (3) gives,

$\therefore$ By SSS similarity $\Delta ADB \sim \Delta DGF$ (by

$\Rightarrow \angle DAB = \angle FDG = \alpha$

And $\angle ABD =$ ∠DFG ⇒ ABDF is a cyclic quadrilateral. [$\because \angle ABD$ = exterior angle $\angle DFC$]

∴ EA & EF are secant to the circle.

ED x (ED+DF) = EB (EB+BA)

ED x (ED+5) =2[12]

$ED^2 + 5ED^2 - 24 = 0$

(ED-3) (ED+8) = 0

⇒ED = 3 as (ED can't be –ve)

In Δ EAF, BC is transversal

$$\frac{EB}{AB} \times \frac{AC}{CF} \times \frac{FD}{DE} = 1$$

$$5\,\frac{2}{10} \times \frac{2X}{CF} \times \frac{5}{3} = 1$$

$\mathrm{CF} = \frac{2x}{3}$

$\mathrm{AF} = \mathrm{AC\text{-}CF}$

$= 2x - \frac{2x}{3}$

$= \frac{6x-2x}{3} = \frac{4x}{3}$

$\frac{AF}{CF} = \frac{\frac{4x}{3}}{\frac{2x}{3}} = \frac{4}{2} = 2$

$\therefore \mathrm{AF} = 2\mathrm{FC}.$

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